## Buddhism and Quantum Mechanics

The concept of "indistinguishability" is fundamental to both the Buddhist theory of Sunyata, or Emptiness, and the scientific theory of Quantum Mechanics. I will use this insight to show that Quantum Mechanics is a consequence of the Buddhist doctrine of "The Two Truths."

The most basic doctrine of Buddhism is called The Two Truths. The UltimateTruth is that nothing has inherent existence. The Relative Truth is that in our everyday human world, everything acts as if it had inherent existence. The doctrine that nothing has inherent existence, the Ultimate Truth, is also called Sunyata or Emptiness. In Mahayana Buddhism, completely understanding the meaning of Emptiness, this lack of inherent existence, along with developing Compassion, is said to take 3 long eons (which are each longer than the age of the present universe). These are called the Two Accumulations and their accomplishment is the path to Enlightenment.

What does Inherent Existence mean, why is it so difficult to completely understand, and why would this understanding (along with developing compassion) lead to enlightenment?

Where to start? If you see a rock in front of you, that rock seems to exist independently of any context and seems to be distinguishable from what is "not that rock". Surely that rock exists independently of whether I "observe" it or not, and it has certain properties independently of whether I observe these properties or not. It is just there and I can look at it or not look at it. This particular rock is not "inherently" special apart from the fact that I am observing it or that I consider it to be special. Also, something either happens or not, independently of whether I see it happen or know that it happened. The rock either fell over or it did not fall over. That is, in our minds, the way we perceive the world. In the "world itself", the "Welt-an-sich", there is no way to distinguish anything from anything else. It simply is. The Buddhists call this the "formless" realm and give it the name Dharmakaya. Each living creature interacts with the world with senses that have developed over evolutionary time to enable its ancestors to survive and reproduce. The world revealed to the creature by these senses, the everyday world of the creature, is said to be the "umwelt" of the creature. The form of the Two Truths given above, where the Relative Truth is that in our human umwelt, everything acts as if it had inherent existence, might be considered the Theravatic Buddhist version of the Two Truths. The Mahayana Buddhist version of the Two Truths is then that the Relative Truth is that in each living creature's umwelt, everything acts as if it had inherent existence.

Living creatures have evolved many different ways of sensing their environments. These often involve vibrations in the surrounding medium e.g. hearing and seeing. Mostly "seeing" is restricted to a single octave of the electromagnetic medium (where most of the energy of the solar radiation is). Of course, that is our human specialty. Many creatures mostly sense using surface vibrations. An interesting example of this is the spider that creates its own "spider web" and senses the vibrations of this web. We might consider that humans in the last few centuries have created their own "human web" of scientific instruments to extend their "seeing" into the whole electromagnetic medium and to vastly increase the sensitivity of this seeing. Humans now live, perhaps a bit uneasily, in two umwelts: the human umwelt that we evolved into and the scientific human umwelt that is evolving from our scientific endeavors. The human umwelt has many peculiarities, as seen from the perspective of the scientific human umwelt, that result from
its evolutionary origin. For example, looking straight down 100 feet and looking to the side 100 feet does not seem remotely the same, or a "rock" in front of us is solid to our senses but in the scientific view is a bunch of molecules that are overwhelmingly made up of empty space. This lack of correspondence between the human umwelt and the scientific human umwelt is sometimes used as an example of the interpretation of the Buddhist Theory of Sunyata that "things don't exist in the way that they seem to".

The question that we want to consider is "Do the Two Truths hold for the scientific human umwelt?" If not, is there some modified version of the Two Truths that does hold?

A creature's umwelt is built up by how its evolutionary ancestors successfully sensed their environment to survive and reproduce. This led to a particular body, behavior, and, eventually, a mind filled with "qualia". In the everyday umwelt of the creature, there is no distinction between a quale and what it "represents". However, in the human scientific umwelt, there is such a distinction. Consider, for example, the old question "When a tree falls in a forest and no one is there to hear it, does it make a sound?" If by a sound, you mean a quale in a mind, then the answer is no, but if you mean a pressure wave in the air, then the answer is yes. While in the ordinary human umwelt, there is only one thing, the sound, in the scientific human umwelt there are two things, a quale in a mind and a pressure wave in the air.

Our modern scientific instruments allow us to "see" things that are much smaller than any living being, things that are not part of any living being's umwelt. We have not evolved qualia to represent these "things". However, we have developed a purely mathematical way of understanding the behavior of the world at these small scales, which we call Quantum Mechanics. Neils Bohr developed a philosophical foundation for Quantum Mechanics which is part of what is called the Copenhagen Convention. We have said that part of what is meant by inherent existence is that things seem to exist outside of any context. Bohr noted that, in a fixed context, everything acts as if it had inherent existence in the sense that a true statement has the property that its opposite is false i.e. the truth and falsity of statements are distinguishable. He thought that a deeper understanding required the simultaneous use of two contexts that are mutually contradictory. Then a "deep" truth would have the property that its opposite was also a deep truth. These two mutually contradictory contexts are said to be "dual". The canonical example in Quantum Mechanics is wave/particle duality. The canonical example in Buddhism might be considered to be Compassion/Emptiness duality. Consider Umwelt/(Welt-an-sich) duality. Neither an umwelt or the Welt-an-sich has inherent existence. One can only say something about the Welt-an-sich, even if it exists, in terms of an umwelt. In the scientific view, the Welt-an-sich exists as consistency in the umwelt. It is tempting to think that the rock I see in front of me in my umwelt, that is, as a quale, "corresponds" to or "labels" a rock or something in the "actual" world, the Welt-an-sich. This, however, only makes sense in the umwelt, where everything is distinguishable. It makes no sense in the Welt-an-sich, where nothing is distinguishable. Considering this Umwelt/(Welt-an-sich) duality highlights the basic role played by distinguishability.

We have mentioned that the scientific human umwelt is based on having radically increased the human ability to observe the world. An equally important ingredient in the scientific human umwelt is the human ability for what is called "hypothetical thinking". These are thoughts of the form "If ...., then ...". Is such a thought just gibberish or does it make some kind of sense? If it
makes sense, is it true or false, and what would that mean? One (psychological) interpretation of the Two Truths might be that all of our thoughts in our daily life are "hypothetical thoughts", but the "If" part is implicit. It is completely left out of our conscious thought. In other words, the (Psychological) "solidity" of our daily world is based on the fact the "context" of our thought is unexamined and implicit so that our (psychological) daily world seems to exist independently of any context. Under these circumstances, it is difficult for different individuals to agree on what is gibberish and what "makes sense", and what is "true" and what is "false".

The ancient Greeks at the time of Euclid came up with the idea of isolating a small part of our thoughts and being quite clear that these were hypothetical thoughts. The "it" parts were called "axioms" and "postulates" and the "then" parts were called "Theorems". In the modern way of looking at things, all of mathematics is hypothetical thinking. Further, to the mathematician, whether the statement "If..., then..." is true or false is completely determined by logic.

Another type of hypothetical thought, used by physicists, is the "Gedanken Experiment". Here the "If..., then..." is a conjecture about what happens in the world. The truth or falsity is determined by logic and by consistency with what is known, or believed to be known, about the world. Sometimes, as in the famous Gedanken Experiments of Albert Einstein, these hypothetical thoughts show that some deeply ingrained belief about the world is false e.g. that time and space exist inherently and independently of each other.

The bedrock of modern science is the determination of the truth of a hypothetical statement by an actual experiment in the world. However, it is a little bit more than this. To show the truth of a statement of the form "If..., then...", by an experiment, the experiment must be repeatable. There are two reasons for this. The first is social. How can one extend what everyone can agree upon beyond mathematics? In modern science, you don't have to agree with me because I say so, you can do the experiment yourself. At a deeper level, something that happens once could have happened by accident. The more an experiment is repeated, the more likely it was not accidental, but was the result of the truth of an "If..., then..." statement.

Last but not least, hypothetical statements let the scientist make up "theories" about how the world works.

I am going to present a version of the Buddhist Doctrine of the Two Truths for the Scientific Human Umwelt and argue that, at the most basic level, this is the same as the scientific theory of Quantum Mechanics. However, the context of Buddhism is the Buddha's desire to alleviate suffering i.e. compassion, and the context of Science is the desire to understand the world i.e. curiosity, so beyond this basic level each develops separately. Hopefully, the viewpoint that I present will help clarify a few things that are often confusing and confused.

We perceive the world both in terms of objects that have properties and in terms of actions or happenings. I will first examine how an object with a property "acts like it has inherent existence".

In traditional Tibetan Buddhist practice, they might begin by contemplating the lack of inherent existence of a pot. The focus of their meditation would probably be on how the "distinguishability" of what is the pot and what is not the pot is not inherent in the pot but rather in the "distinguisher". In our formulation, this means that the distinguishability lies in the umwelt, not the Welt-an-sich. Beyond that, if the pot has the property of, say, being blue, then it has this
property independently of whether I observe the pot or not, or whether I know anything about the pot. In my umwelt, this property of being blue seems to be inherent in the pot.

Let's look a little more closely at our umwelt and how it arose. An organic being is an almost closed metabolism that needs to interact with its environment to keep its entropy from increasing and hence dying. Suppose that such a set of entities can reproduce. If these entities interact with their environment in two different ways that lead to different probabilities of surviving and reproducing, then, in future generations, the behavior leading to greater reproduction will be more prevalent in the population. That is, the individual will distinguish between the two behaviors by behaving more often in the way that leads to greater reproduction. This means that, over time, the ability to "distinguish" gets built into the individuals of the species. While in the Welt-an-sich nothing is distinguishable, in the context of an individual of such an evolving species, things are distinguishable. You live entirely in the context of your own being, that is, in your own human umwelt, and, within this context, everything is distinguishable. At a basic level, we may view each interaction of a being with its environment as being an observation. Built into the way that I am talking is the way in our umwelt that we divide an "observation" into two parts: an "observer", "me", and "what is observed". (A central practice in Buddhism is the attempt to deconstruct this innate construction of an " 1 ".) We then view this "what is observed" as "being" the world, that is, as existing inherently and independent of being observed as part of my umwelt. We may view this world as composed of "objects" with "properties" or as "things happening". We do this both in our human umwelt and in our scientific human umwelt.

## A MATHEMATICAL MODEL OF THE TWO TRUTHS

We start with a model using Umwelt/(Welt-an-sich) duality and later extend this to a model using (Scientific Human Umwelt)/(Welt-an-sich) duality.

The basic object in our model is an "observation". An observation is an event. In particular, this means that, given two observations, the first observation occurs either before, simultaneously with, or after the second observation. An observation can be repeated and an observation can be observed. An observation can not affect the result of an observation that comes before it. Relative to a given observation, observations that are before this observation are in the "past" and are fixed i.e. they either happened or they didn't happen. Observations that are after this observation are in the "future" and are not fixed i.e. one can choose either to make or not make such an observation.

The "result" of an observation is picking out one of a set of possible values for the observation. A subtlety here is that the results of an observation are only distinguishable if there is an observation of these results. This means that an observation that is not observed i.e. whose results are not observed, can not affect the result of any future observation. The phrase "the observation $\Phi$ gave the value P " means that the observation $\Phi$ was followed by an observation that distinguished the value P from the other possible values for $\Phi$.

I will also use the phrases "immediately after" and "immediately before". An observation comes immediately after a second observation if and only if the second observation comes
immediately before the first observation. Also, if an observation follows immediately after an observation that follows immediately after another observation, then the first observation follows immediately after the last observation i.e. the relation "immediately after" is transitive.

## HOW THE WELT-AN-SICH EXISTS

You can look out and see the world and think that that is the world. That is the way we normally think and it is quite difficult to think otherwise.

You can look out and think that this is not the actual world. The actual world is what is seen with our powerful scientific instruments and theories e.g. atoms and fields.

You can look out and think that this is not the actual world and also what the scientists see is not the actual world, but what I see represents or corresponds to something in an actually existing world.

You can look out and think this is all I have. All that I can say is that the consistency in my Umwelt is evidence that some kind of external world exists.

Finally, you can look out and think that the theory of Sunyata or Emptiness, the Ultimate Truth, says that things don't exist in the way that they seem to exist. Consistency in my Umwelt is not "evidence" that the Welt-an-sich exists, it is "the way" that the Welt-an-sich exists.

In our Model, the basic manifestation of this consistency, that is the way that the Welt-an-sich exists, is the following:

PERSISTENCE (1): If the observation $\Phi$ gives the value $P$, then, if $\Phi$ is immediately repeated, it will give this same value $P$.

DEFINITION OF A STATE: Say that something is in the state $|\mathrm{P}\rangle$ if the hypothetical statement "if I make the observation $\Phi$ I will obtain the value $P$ " is true.

PERSISTENCE (1'): If the observation $\Phi$ of an object gives the value $P$, then immediately after the observation the object will be in the state $|\mathrm{P}\rangle$.

Imagine a blue pot in front of you. The blueness of the pot seems to exist independently of yourself or your seeing the pot. However, in your Umwelt, the only meaning this can have is that the pot is in the state |blue $\rangle$.

PERSISTENCE (2): If an object is in the state |P>, then immediately afterward it is also in the state $|\mathrm{P}\rangle$.

PERSISTENCE (2'): If an object is in the state |P>, if it is observed immediately afterward, the value $P$ will be obtained.

PERSISTENCE (2"): If an object is observed and the value $P$ is not obtained, then the object was not in the state $|\mathrm{P}\rangle$ immediately before the observation.

These two forms of the assumption PERSISTENCE are similar but not equivalent.

## THE RELATIVE TRUTH

The Relative Truth says that everything in the Umwelt acts like it has inherent existence and, in particular, is distinguishable. In our theory, this means distinguishable by an observation. Along with observations, the fundamental objects in our theory are the states of the form $|\mathrm{P}\rangle$. If we want to know if something is in the state $|\mathrm{P}\rangle$, we can observe it. If we don't observe the value P, we know by PERSISTENCE 2" that it was not in the state $|\mathrm{P}\rangle$ immediately before the observation. If we observe the value P , we need to be able to conclude that it was in the state $|P\rangle$ immediately before the observation to be able to conclude that the state $|\mathrm{P}\rangle$ is distinguishable.

DISTINGUISHABILITY: The state $|\mathrm{P}\rangle$ is distinguishable means that if an observation of an object gives the value $P$, then the object was in the state $|P\rangle$ immediately before the observation was made.

THE RELATIVE TRUTH: In an Umwelt, every state $|\mathrm{P}\rangle$ is distinguishable i.e. an observation of an object giving the value P implies that the object was in the state $|P\rangle$ immediately before the observation.

## GEDANKEN EXPERIMENTS

The essence of a scientific experiment is that it is repeatable. An observation that is observed and has the result $P$ can be thought of in two different ways:

PREPARATION: An observation that gives the result P can be thought of as a "Preparation" of the state $|\mathrm{P}\rangle$. Our Gedanken Experiments will begin with a Preparation. We are thus assuming that this Preparation can be repeated indefinitely.

RECORDING: An observation that gives the result P can be thought of as "recording" the value P. Our Gedanken Experiments will end with a Recording.

GEDANKEN EXPERIMENT: Our basic Gedanken Experiment consists of repeating many times a given Preparation followed by a given Recording. For each possible value $R_{i}$ of the observation that is the Recording, the number of times the experiment gives the value $R_{i}$ is recorded and these numbers are divided by the total number of times the experiment is run. The result is a set of numbers $r_{i}$ corresponding to each $R_{i}$. These numbers satisfy
$\sum r_{i}=1$ i.e. the sum of the $r_{i}$ equals 1 . We assume that our Gedanken Experiment is run in such a way that, to any given degree of accuracy, repeating the experiment will give the same values of the $r_{i}$. In particular, it is assumed that if the Recording comes immediately after the Preparation, then this is a valid Gedanken Experiment i.e. is repeatable with the same results.

PERSISTENCE(1) can be viewed as a Gedanken Experiment. It says that if the same observation is used for both the Preparation and the Recording, the Recording comes immediately after the Preparation, and the state $|\mathrm{P}\rangle$ is prepared, then $r_{i}=1$ for the $r_{i}$ corresponding to the value $P$ and $r_{j}=0$ for all other values of the observation.

This allows me to clarify what I mean by "immediately". We tend to blink around 12 times a minute and each blink lasts around $1 / 3$ of a second. If "the world" was not appreciably the same at the end of this $1 / 3$ of a second as it was at the beginning, then we would hardly survive and reproduce. Actually, it is much more intense than that. Almost all of what you are seeing "now" is created by your mind using information from what you saw in the past. That you do not perceive any change in the $1 / 3$ of a second while you blink does not mean that there was no change. It means that the probability that there was change that would negatively affect your survival and reproduction was very low. So low, that evolutionary forces could not cause your "reaction time" to be shorter. PERSISTENCE means that, as the time between two observations goes to 0 , the probability that the two observations will be the same goes to 1 . The use of the phrase "immediately" is meant as an attempt to keep technical details to a minimum in the interest of readability. However, note that the object may not be in the state $|\mathrm{P}\rangle$ after the observation at all. The ancient Greeks were quite bothered by this kind of thing e.g. Zeno's Paradox, but since the invention of the Calculus with infinitesimals or limits, we tend to take this kind of thing in stride.

PERSISTENCE (3): If our Gedanken Experiment starts with the Preparation of a state $|P\rangle$ and is immediately followed by a Recording which is not the same observation as the Preparation, then a unique set of numbers $r_{i}$ are obtained that record the fraction of the recorded values equal to $R_{i}$, where the $R_{i}$ are the possible values of the observation that is the Recording. Then $\sum r_{i}=1$. Here unique means that if the Gedanken Experiment is repeated, the same values $r_{i}$ will be obtained.

## TRADITIONAL TIBETAN BUDDHIST STUDENTS

Traditional Tibetan Buddhist students might begin their study of Sunyata or Emptiness by contemplating the non-inherent existence of a pot. They would examine in great detail the many ways that the pot is not distinguishable. This is difficult and subtle because, in our everyday world, our umwelt, the pot acts like it had inherent existence i.e. it is distinguishable. Just observe it! The traditional way would be to deconstruct this observation, the observer, and what is observed, and follow this down a rabbit hole that never seems to end. In some religious traditions, they climb out of this rabbit hole by deciding that the "Observer" has inherent existence. In some they decide there is an "ÜberObserver", that is, "God". Not in Buddhism. They are content to fall down the rabbit hole for three long eons
if that is what it takes．We will not follow these Buddhist students down this rabbit hole but rather consider a possibility not available to them．Namely，we will consider the hypothetical possibility of a＂pot＂in the Scientific Human Umwelt that does not＂act like it had inherent existence＂i．e．that the Relative Truth does not hold for the Scientific Human Umwelt．

What would the world look like if the Relative Truth failed？We will consider， hypothetically，a world where the Relative Truth fails，the Ultimate Truth holds，and an observation may be either observed or not observed．This turns out to be the world of Quantum Mechanics．

Consider in your thought，or Gedanken，a pot．Perhaps a pot that is red or blue．Rather than considering the lack of inherent existence of the pot，we will consider the lack of inherent existence of the pot being red or blue．In our language，we are considering the lack of inherent existence of the states｜red $\rangle$ and｜blue $\rangle$ ．

## SCHRÖDINGER＇S CAT

Suppose we have a pot that when observed，is either red or blue，but these states do not act like they had inherent existence i．e．if we observe the pot and see red，we can not conclude that the pot was in the state｜red $\rangle$ before the observation and if we see blue， we can not conclude that the pot was in the state｜blueो．Remarkably，by PERSISTENCE（2＂）， if we observe the pot to be red，then we can conclude that the pot was not in the state ｜blue〉 before the observation and，if we observe the pot to be blue，we can conclude that the pot was not in the state $|\mathrm{red}\rangle$ before the observation．This means that if the pot had to be in either the state｜red〉 or｜blue〉 before the observation，then we could conclude that， an observation of red means it was not in the state｜blue〉 before the observation and hence it was in the state｜red $\rangle$ and similarly for blue and the state｜blue〉．This means that， if the states $\mid$ red $\rangle$ and｜blue $\rangle$ are not acting like they had inherent existence，then there must be a pot that is not in either the state｜red〉 or｜blue〉 but which is always seen to be either red or blue when observed．Tibetan students of Buddhism are perfectly happy considering colored pots but Western students need something more exciting．For this reason，Erwin Schrödinger used a cat rather than a pot and the two－valued property alive or dead for his Gedanken Experiment．If the cat is observed and found to be dead，then you have killed the poor thing because it was not in the state｜dead〉 before the observation．（A policeman stops Schrödinger＇s car and searches the trunk（bonnet）for contraband．He finds a dead cat． He says to Schrödinger＂Sir，there is a dead cat in your trunk（bonnet）．＂＂Schrödinger replies ＂Yes，NOW there is！＂）Such a cat is said to be a Schrödinger＇s Cat．With time，any object that is not in either the state $|\mathrm{P}\rangle$ or $|\mathrm{Q}\rangle$ for a property that takes on precisely the two values $P$ and $Q$ ，has come to be known as a Schrödinger＇s Cat．

Schrödinger＇s popularization of his cat has led to popular confusion in two ways． First，no cat in the everyday world，the human umwelt，is a Schrödinger＇s cat．More subtly，in our umwelt，being dead and being in the state｜dead）are the same，but they are not the same for Schrödinger＇s cat．The crucial difference is that one can tell if a cat is dead
by observing it, but you can't tell if Schrödinger's cat is, or isn't, in the state |dead> by observing it. For this reason, I have used the somewhat clunky phrase "is in the state $|\mathrm{P}\rangle$ " to mean that the phrase "if an observation is made the value $P$ will be obtained" rather than "has the property P". Also, this corresponds to the usage in Quantum Mechanics, where this "ket" $|\mathrm{P}\rangle$ was introduced by Paul Dirac.

Schrödinger's Cat:
If an object with a two-valued property is not acting like it has inherent existence, then there is such an object that is a Schrödinger's Cat. i.e. if the property values are $P$ and $Q$, there is an object that is not in either of the states $|\mathrm{P}\rangle$ or $|\mathrm{Q}\rangle$.

We will use the following principles:

OCCAM'S RAZOR: When solving a problem or developing a theory, try the simplest possibility first. Assume this is correct until a contradiction is found.

LACK OF INHERENT SPECIALNESS: Sunyata says that nothing has inherent existence. In particular, specialness does not have inherent existence i.e. nothing is inherently special.
[ Galileo's Principle of Inertia says that an object on which no force acts travels in a straight line at a constant speed i.e. with constant velocity, where velocity is a vector that encodes both the speed and direction of motion. With the calculus at our disposal, we can state an "instantaneous" version of Galileo's Principle of Inertia as "the force F acting on an object is 0 precisely when the rate of change of the velocity, i.e. the acceleration $A$, is 0 ". The simplest relationship between $F$ and $A$ which has them both 0 at the same time is when they are proportional i.e. F = mA for someconstant $m$. This is Newton's famous Law of Motion. For two centuries no contradiction to anyobservation was found for Newton's Law of Motion and so by Occam's Razor, it was assumedto be correct. In 1906, Einstein applied the principle of the Lack of Inherent Specialness tothe set of inertial coordinate systems to conclude that at high velocities Newton's Law of Motion fails. (Einstein also used the fact that the speed of light is finite. By the principle of the Lack of Inherent Specialness of inertial coordinate systems, light must travel at the same finite speed in every coordinate system. But if in one coordinate system $F=m A$, if $I$ undergo a constant force, I will undergo a constant increase in velocity, and so will eventually be going at the speed of light. At that point, in the inertial coordinate system co-moving with me, light can not have a nonzero constant velocity.)]

## TYPES OF SCHRÖDINGER'S CATS

TAME SCHRÖDINGER'S CATS: Say that a Schrödinger's cat is "tame" if it can be prepared.

WILD SCHRÖDINGER'S CATS: Say that a Schrödinger's cat is "wild" if it is not tame i.e. it can not be prepared.

To be able to perform Gedanken Experiments on our Schrödinger's cat we need one that is tame. We will invoke Occam's Razor to assume that a tame Schrödinger's cat exists.

SCHRÖDINGER'S CAT (1): A tame Schrödinger's cat exists.
Let $\Phi$ be our original observation that prepares the states $|\mathrm{P}\rangle$ and $|\mathrm{Q}\rangle$. Let $\Psi$ be an observation that prepares a Schrödinger's cat i.e. an object that is not in the state $|\mathrm{P}\rangle$ or $|\mathrm{Q}\rangle$. We can run our Gedanken Experiment using $\Psi$ to prepare the Schrödinger's cat and immediately follow with the observation $\Phi$. This will give us two numbers, $p$ and $q$, with $p+q=1$, where $p$ is the proportion of the time that our experiment gives the value $P$ and $q$ is the proportion of the time that it gives the value $Q$. By assumption $0<p<1$ and $0<q<1$.

Since no such pair of numbers is "inherently special", we can use LACK OF INHERENT SPECIALNESS to conclude (or use OCCAM'S RAZOR to assume) that a Schrödinger's cat exists for each such pair p,q. Call this a (p,q) Schrödinger's cat or a Schrödinger's cat of type ( $p, q$ ).

SCHRÖDINGER'S CAT (2): A tame Schrödinger's cat of type ( $p, q$ ) exists for each pair $0<p<1,0<q<1$ with $p+q=1$.

GENERALIZATION: Let $\Phi$ take the values $\mathrm{P}_{\mathrm{i}}$ and so prepare the states $\left|\mathrm{P}_{\mathrm{i}}\right\rangle$. If the states $\left|P_{i}\right\rangle$ don't act like they have inherent existence, then for each set $\left\{r_{i}\right\}$ with $\Sigma r_{i}=1$, there is an observation $\Psi$ that prepares a state $|\mathrm{P}\rangle$ which, in a Gedanken Experiment starting with $|\mathrm{P}\rangle$ and ending immediately after with the observation $\Phi$, gives the data $\left\{r_{i}\right\}$ i.e. the value $P_{i}$ is obtained with probability $r_{i}$.

## WILD EINSTEINEAN SCHRÖDINGER CATS, ENTANGLEMENT, SPOOKY ACTION AT A DISTANCE, AND THE NO CLONING THEOREM

Consider that the "object" in front of you is a set of two pots, each of which can be either red or blue. An observation of this object is an observation of each pot separately and has four values (red,red), (red,blue), (blue,red), and (blue,blue). If the states prepared with these values do not act like they have inherent existence, then, by GENERALIZATION, there will be an "observation" of the two pots that prepares a state $|\mathrm{P}\rangle$ that, when observed by "looking at the pots to see their color", has zero probability of obtaining either (red,red) or (blue,blue), and non-zero probability of obtaining (red,blue) and (blue,red).

## EXISTENCE OF A WILD SCHRÖDINGER'S CAT

In this state $|\mathrm{P}\rangle$, each of the pots, considered individually, is a wild Schrödinger's cat:
Suppose these two cats are tame with $r_{1}$ and $b_{1}$ being the probability that the first pot will be observed to be red or blue, and $r_{2}$ and $b_{2}$ being the probability that the second pot will be observed to be red or blue. Then the probabilities for the observation of the two pots in the state $|P\rangle$ are $r_{1} r_{2}$ for (red,red), $r_{1} b_{2}$ for (red,blue), $b_{1} r_{2}$ for (blue,red), and $b_{1} b_{2}$ for (blue,blue).
Then $r_{1} b_{2} \neq 0$ implies that $b_{2} \neq 0$, and $b_{1} r_{2} \neq 0$ implies that $b_{1} \neq 0$. Thus $b_{1} b_{2} \neq 0$, contradicting the assumption that the probability of obtaining the value (blue,blue) was zero.

## ENTANGLEMENT

These two wild Schrödinger cats are said to be entangled. When a system that is in a preparable state, i.e. is tame, is composed of two subsystems that are not preparable, i.e. are wild, then the two subsystems are said to be entangled.


EINSTEINEAN CATS AND SPOOKY ACTION AT A DISTANCE

These two entangled wild Schrödinger cats have an interesting property. If the first cat is observed and is seen to be red, then, since the probability of
getting an observation of (red,red) is zero, it must be that if the second pot is observed, the value red will not be obtained and hence the value blue will be obtained. Similarly, if the first pot is observed to be blue, then, if the second pot is observed, it will be red.

Einstein was fascinated and disturbed that the second pot could be far away, perhaps even millions of light years away (at least in our Gedanken Experiments). In a famous 1936 paper, he called this "Spooky Action at a Distance". Something happens here (the first pot is observed to be, say, red), and instantly something happens there (perhaps millions of light years away, a pot changes from being a wild Schrödinger's cat into being in the state |blue $\rangle$ ).

Returning to considering hypothetical cats rather than pots and calling our entangled cats Einsteinian cats, if Einstein observes his cat and she is alive, he has killed his other cat even though she is millions of light years away. She was a wild cat before Einstein's observation and so not in the state |dead〉 but after the observation she was in the state |dead〉. Poor kitty! This bothered Einstein very much

People often take this "something happens here and immediately something happens there" to mean AHA, THE WORLD IS CONNECTED. However, not so fast.

## SPECIAL RELATIVITY

We started our discussion by saying that an observation was an event and interpreted this to mean that any observation was either before, simultaneous with, or after a given observation. This is true in a natural Umwelt, the daily life of a creature. However, this is not true in the Scientific Human Umwelt. This was Einstein's world-shaking discovery in his 1906 paper on Special Relativity. He realized that this was a consequence of the fact that no inertial coordinate system was inherently special (even though coordinate systems that are comoving with me, or the earth, or the sun, are certainly special to me). He defined what he called the "lightcone" of an event. The future light cone consists of all points of space-time (events) that could be the arrival of a light ray emanating from the original event. The past light cone consists of all events that could be the emanation of a light ray that the original event is the reception of. The interior of the future light cone consists of all events that can be reached from the original event moving at less than the speed of light. The interior of the past light cone consists of all events that can be connected to the given event traveling at less than the speed of light. The exterior of the light cone consists of everything else. Inertial observers are observers moving at constant velocity.

Einstein showed that the future light cone and its interior are precisely those events that all inertial observers agree are after the given event and the past light cone and its interior are precisely all events that all inertial observers agree are before the given event. For each event that is outside of the light cone, there is an inertial observer that will see this event as happening before the given event, there is an inertial observer that will see this event as being simultaneous with the given event, and there will be an observer that will see this event as happening after the given event. Also, an event can only affect an event that lies in its future time cone. It can not affect an event that is in its past light cone or an event that is outside its light cone.

Suppose that Einstein observes his first cat and she is alive. Then, for every point (event) on
the world line of his second cat that lies outside of the light cone of this observation，there will be an inertial observer who sees this event as being simultaneous with Einstein＇s observation of his first cat．Thus this is the moment when＂something happens＂ i．e．the second cat goes from being a wild Schrödinger cat into being in the state｜dead〉． But，how can this be？If the second Einstein cat is millions of light years away，then this would mean that every moment for millions of years is＂the moment＂when＂something happens＂．It can only mean that it is physically impossible，outside of the light cone of Einstein＇s observation，to tell if this＂something happened＂．

In our theory we have only assumed that you couldn＇t determine if Einstein＇s second cat was in the state｜dead〉 by observing her．We now see that it is impossible to determine if she was in the state｜dead $\rangle$ by any means at all．

An interesting way to try to determine if Einstein＇s cat was in the state｜dead〉 would be to find a way to＂clone＂her without observing her．You can＇t use Einstein＇s cat for a Gedanken Experiment because a Gedanken Experiment has to be repeatable and after observing Einstein＇s cat she is no longer Einstein＇s cat．Being able to＂clone＂Einstein＇s cat without observing her means being able to run a Gedanken Experiment with the＂clones＂as a proxy for Einstein＇s cat．One could then run Gedanken Experiments with the clone being observed with different observations and values for these observations．Eventually，you would find an observation and a value $P$ that gives a probability of finding an observation of the clone with value P to be＂arbitrarily close＂to 1 （to within the＂experimental accuracy＂of our Gedanken Experiment）．The clone was thus in the state $|\mathrm{P}\rangle$ ．This means，by assumption，that the original cat was in the state $|P\rangle$ ．If there is no such observation and value $P$ ，then the original cat was a wild Schrödinger＇s cat．

NO CLONING THEOREM：An unknown Schrödinger can not be cloned without destroying the original．

Interestingly，it turns out that you can clone an unknown cat if you are willing to destroy the original．Since the new clone can be at a distance from the original，this is rather dramatically called＂quantum teleportation＂（for example，the original could be here and the new clone on the Holodeck of your orbiting spaceship）（occasionally in the news there are reports of new＂distance records＂for quantum teleportation）．

Suppose a coin is flipped in the air，caught in the hand，and placed on the opposite wrist with the hand covering the coin．The probability of heads being up is $1 / 2$ ．If you＂call＂heads while the coin is in the air，this is a property of the coin．It might wind up with either heads up or heads down with equal probability．However，if you call heads after the coin is on the wrist， then the indeterminacy，the probability $1 / 2$ ，is a property of your ignorance．Whether the coin is heads up or down is already fixed．Suppose the coin was fake，having both sides the same．Then your odds are still $1 / 2$ but，if you called when the coin was in the air，this is now purely a property of your ignorance．Suppose that while the coin was in the air，someone was able to change a fair coin into a biased coin that had both sides the same． If you called in the air，the odds would still be this $1 / 2$ but the＂something happened＂would be that the odds changed from being a property of the coin to being a property of your
ignorance.
Suppose someone wanted to observe Einstein's second cat. Suppose, before he made the observation, he wanted to guess whether he would observe a dead cat or an alive cat. If the cat was still in the entangled Einstein state, then whether his guess was correct or not was a property of the cat. No matter what he guessed, he might be wrong. When Einstein observes his first cat, the second cat will be in the state |dead〉, and so whether his guess is right or wrong is purely a matter of his ignorance. Put slightly differently, if he observed "dead", then after the observation the probability that this observation would have given the value dead is 1 . Before the observation, it was less than 1 if she was still a wild Schrödinger's cat and equal to 1 if she was in the state |dead $\rangle$. This probability is always 1 after the observation and so says nothing about what the probability was before the observation. When "something happens here", Einstein observes his cat, the "something happens there" is that the probability of an observation of the second cat giving a given value changes from not being equal to 0 or 1 to being either 0 or 1 i.e changes from being indeterminate to be determinate. When Einstein observes his first cat, he knows that the change has happened to his second cat. Being outside of the time cone of Einstein's observation, it is impossible for the person examining the second cat to know if this change has occurred.

## SOME GEDANKEN EXPERIMENTS

We used the principle of LACK OF INHERENT SPECIALNESS to argue that if a single tame Schrödinger's cat exists, then Schrödinger's cats of all types ( $r, s$ ) with $r+s=1$ should exist. Let $\Phi$ be our original observation with values $P$ and $Q$. Let $\Psi$ be an observation with a value $R$ that gives a Schrödinger's cat of type $(r, s)$ with $r+s=1$. This means that if we run a Gedanken Experiment starting with the preparation of the state $|\mathrm{R}\rangle$ using $\Psi$ immediately followed by the observation $\Phi$, then the probability of getting the value $P$ is $r$ and the probability of getting the value $Q$ is s. Let me introduce some notation to make it easier to keep track of this. Let $|R\rangle \rightarrow P$ or simply $R \rightarrow P$ mean that the state $|R\rangle$ is recorded by $\Phi$ to give the value $P$. Let $[|R\rangle \rightarrow P]$ or $[R \rightarrow P]$ be the probability for this possibility that is found running the Gedanken Experiment. In this notation, $[R \rightarrow P]=r$ and $[R \rightarrow Q]=s$.

Just as no particular Schrödinger's cat is Inherently Special, we would like to argue that $\Phi$ is not Inherently Special compared to the $\Psi$ that define Schrödinger's cats. This means that, relative to $\Phi, \Psi$ defines a Schrödinger's cat, but, relative to $\Psi, \Phi$ defines a Schrödinger's cat. Moreover, these two possibilities should not be distinguishable by running a Gedanken Experiment. This means, first of all, that $\psi$ must have just two values, say, $R$ and $S$. Say that the Schrödinger's cat $|S\rangle$ is "complementary to $|R\rangle$. There are four possible Gedanken Experiments that we can run. We can prepare $|\mathrm{R}\rangle$ or $|\mathrm{S}\rangle$ with $\Psi$ and immediately follow with the observation $\Phi$, or prepare $|P\rangle$ or $|Q\rangle$ with $\Phi$ and immediately follow with the observation $\Psi$. All four of these Gedanken Experiments should result in the same two probabilities $r$ and $s$. More specifically, we will have

$$
\begin{aligned}
& r=[R \rightarrow P]=[P \rightarrow R]=[S \rightarrow Q]=[Q \rightarrow S] \\
& s=[R \rightarrow Q]=[Q \rightarrow R]=[S \rightarrow P]=[P \rightarrow S]
\end{aligned}
$$

This means that if $|R\rangle$ is of type $(r, s)$, then $|S\rangle$ is of type ( $s, r$ ), and, if $|P\rangle$ is considered to be a Schrödinger's cat relative to $\Psi$, then it is of type ( $\mathrm{r}, \mathrm{s}$ ).

We are now ready to run a more interesting Gedanken Experiment.
First, prepare $|\mathrm{R}\rangle$ with $\Psi$, then immediately observe with $\Phi$, and then immediately observe with $\Psi$ as a recording. We can do this experiment in two different ways. We can either observe the values of the observation $\Phi$, so that this observation is both a preparation and a recording, or we can not observe the values of the observation $\Phi$, in which case this observation cannot affect the following recording by $\Psi$.

Ф OBSERVED: Here $R \rightarrow R$ can happen in two possible ways, $R \rightarrow P \rightarrow R$ and $R \rightarrow Q \rightarrow R$. Which of these two possibilities happens is distinguished by the observation of $\Phi$, so the resulting probabilities add: $[R \rightarrow R]=[R \rightarrow P \rightarrow R]+[R \rightarrow Q \rightarrow R]$. Also probabilities multiply, so $[R \rightarrow P \rightarrow R]=[R \rightarrow P][P \rightarrow R]=r^{2}$ and $[R \rightarrow Q \rightarrow R]=[R \rightarrow Q][Q \rightarrow R]=s^{2}$. Thus $[R \rightarrow R]=r^{2}+s^{2}<1$. Similarly, $[R \rightarrow S]=[R \rightarrow P \rightarrow S]+[R \rightarrow Q \rightarrow S]=[R \rightarrow P][P \rightarrow S]+$ $[R \rightarrow Q][Q \rightarrow S]=r s+s r=2 r s>0$. As is to be expected $[R \rightarrow R]+[R \rightarrow S]=\left(r^{2}+s^{2}\right)+2 r s=$ $(r+s)^{2}=(1)^{2}=1$.

Ф NOT OBSERVED: Here $[R \rightarrow R]=1$ and $[R \rightarrow S]=0$. This is always different than when $\Phi$ is observed so this can be viewed as a test of whether $\Phi$ is observed or even as a definition. The problem here is to analyze this Gedanken Experiment and try to find an "explanation" for this result. There is said to be "total constructive interference" between the two possibilities $R \rightarrow P \rightarrow R$ and $R \rightarrow Q \rightarrow R$ for $R \rightarrow R$, and "total destructive interference" between the two possibilities $R \rightarrow P \rightarrow S$ and $R \rightarrow Q \rightarrow S$ for $\rightarrow S$.

The subtlety here is that after the observation $\Phi$ but before the observation $\Psi$ there was still the possibility of observing the values of $\Phi$. Then $\Phi$ was a recording and we could have recorded $[R \rightarrow P]$ and $[R \rightarrow Q]$. This means that $[R \rightarrow P]$ and $[R \rightarrow Q]$ are definable and $[R \rightarrow P]+[R \rightarrow Q]=1$. Since an observation can not affect the past, these two probabilities must still be valid even if we don't observe $\Phi$. However, in the experiment, $\Phi$ was not observed so it is impossible to distinguish between $R \rightarrow P$ and $R \rightarrow Q$. This means that the two ways of obtaining the value $R$ at the end of the experiment, $R \rightarrow P \rightarrow R$ and $R \rightarrow Q \rightarrow R$, are indistinguishable. Also, the two ways of obtaining the value $S$ at the end of the experiment, $R \rightarrow P \rightarrow S$ and $R \rightarrow Q \rightarrow S$, are indistinguishable. There is no way to assign a probability to any of these possibilities.

## PROBABILITY AMPLITUDES

We might try to find some new kind of probability that works when possibilities are indistinguishable i.e. that these new probabilities add when there are two indistinguishable alternatives. They should also multiply when two things happen in succession and should somehow tell us what the probabilities are when probabilities are definable. In the spirit of Occam's Razor, we should look for the simplest possible way to do this. The simplest numbers after the real numbers are the complex numbers. We should thus try probabilities with complex values. For historical reasons, complex-valued probabilities are called
probability amplitudes.

## COMPLEX NUMBERS

When you were little a teacher probably told you that you could not subtract 3 from 2 "because 3 is bigger than 2". You might have answered "Humbug". l'll just invent a new number, call it -1 , and say $2-3=-1$. This new number -1 will just add and multiply like the old numbers (satisfy the same rules). There are now a bunch of new numbers, like -3 and -7 . I will call these new numbers "negative numbers".

When you were young a teacher probability told you that you couldn't divide 3 by 2 "because 2 doesn't go into 3 ". You might have answered "Humbug". I'll just invent a new number, call it $3 / 2$, and say 3 divided by 2 is $3 / 2$. Actually, for any two numbers, n and m , l'm gonna invent a new number $\mathrm{n} / \mathrm{m}$ and have n divided by m equal to $\mathrm{n} / \mathrm{m}$. All these new numbers will be added and multiplied using the same rules as the old numbers. I will call these new numbers "Rational Numbers". [Rational from ratio. Get it?]

When you were young a teacher probably told you that you could not take the square root of 2. "Even the Ancient Greeks knew that!" You might have answered "Humbug". I'll just invent a new number $\sqrt{2}$ that has $(\sqrt{2})^{2}=2$. If a and $b$ are rational numbers, then $(a+b \sqrt{2})$ is a new number and these new numbers add and multiply just like the old numbers.
[If you were really smart, you might have thought that, if you think of the rational numbers as lying on a line, with bigger to the right and smaller to the left, then $\sqrt{2}$ is kind of a "hole" or "cut" in this rational number line. Actually, there must be "lots" of these holes in this rational line. For example, for $\pi$ or $1+\sqrt{2}$. If you were really, really, really smart, you might have figured out a way to fill in all these holes in the rational number line. You might have called all these new numbers "Real Numbers". Really!. They would add and multiply just like the old numbers.]

When you were young a teacher might have told you that you couldn't take the square root of -1 "because the square of any number is positive". You might have answered "Humbug". I'll just invent a new number and call it $i$. It will satisfy $i^{2}=-1$. If $a$ and $b$ are real numbers, then (a+bi) is a new number and these new numbers add and multiply just like the old numbers. I'll call these new numbers Complex Numbers and I'll call the new numbers (bi) imaginary numbers. [Imaginary numbers because they are not real numbers, not because they are not real. Get It?]

Probably most people have a problem with imaginary numbers because, after all, "Where are they?". The negative numbers were just to the left of the positive numbers, that is, smaller. The rational numbers were "between" integers. Even $\sqrt{2}$ fits in a hole in the rational numbers. But where are the imaginary numbers? The answer is that, just as we can imagine the real numbers lying on a line, we can imagine the complex numbers
lying in a plane. We imagine this plane as containing the real line. Then the imaginary numbers lie on a line that is perpendicular to the real line and intersects it at the point that is the real number 0 , with $0=0 \mathrm{i}$. The complex numbers $a+b i$ with a constant value $b$ form a line that is parallel to the real "axis" at a distance |b|. The complex numbers a+bi with a constant form a line parallel to the imaginary "axis" at a distance |a|. The "absolute value" of a+bi is its distance to 0 , the point where the real and imaginary axis intersect. The absolute value is written |a+bi| and, by the Pythagorean Theorem, is equal to $\sqrt{ }\left(a^{2}+b^{2}\right)$. Since 0 is special, we also have the special "transformation" of this complex plane which is reflection through 0 . This is given by $(a+b i) \rightarrow-(a+b i)=-a-b i$. Also, since the real axis is special, we have the special transformation of the complex plane given by reflection through the real axis. This is given by $(\mathrm{a}+\mathrm{bi}) \rightarrow(\mathrm{a}-\mathrm{bi})$ and is denoted by *, so that $(a+b i)^{*}=a-b i$. Let $\alpha=a+b i$ and $\beta=c+d i$. Then $\left(\alpha^{*}\right)^{*}=\alpha, \alpha \alpha^{*}=$ $(a+b i)(a-b i)=a^{2}-(b i)^{2}=a^{2}-(i)^{2} b^{2}=a^{2}+b^{2}=|\alpha|^{2}$, and $\alpha^{*} \beta^{*}=(a-b i)(c-d i)=(a c-b d)-(a d+b c) i=$ $((a+b i)(c+d i))^{*}=(\alpha \beta)^{*}$. Also $|\alpha \beta|^{2}=(\alpha \beta)(\alpha \beta)^{*}=(\alpha \beta)\left(\alpha^{*} \beta^{*}\right)=\left(\alpha \alpha^{*}\right)\left(\beta \beta^{*}\right)=|\alpha|^{2}|\beta|^{2}$, so $|\alpha \beta|=|\alpha||\beta|$.

$$
\left(\alpha^{*}\right)^{*}=\alpha \quad|\alpha|^{2}=\alpha \alpha^{*} \quad(\alpha \beta)^{*}=\alpha^{*} \beta^{*} \quad(\alpha+\beta)^{*}=\alpha^{*}+\beta^{*} \quad|\alpha \beta|=|\alpha||\beta|
$$

We might mention that the complex numbers "acting just like numbers" means that they obey the usual rules for numbers e.g. the associative, commutative, and distributive laws.

$$
(\alpha+\beta)+\gamma=\alpha+(\beta+\gamma) \quad(\alpha \beta) \gamma=\alpha(\beta \gamma) \quad \alpha+\beta=\beta+\alpha \quad \alpha \beta=\beta \alpha \quad \alpha(\beta+\gamma)=\alpha \beta+\alpha \gamma
$$

The number $\alpha^{*}$ is said to be the "complex conjugate" of $\alpha$.
How might we use these complex numbers to define probability amplitudes?
First of all, we have that $[P \rightarrow R]=[R \rightarrow P]$ and $[P \rightarrow Q]=[Q \rightarrow P]$ when everything is distinguishable. Denote by $(P \rightarrow R),(R \rightarrow P),(P \rightarrow Q)$, and $(Q \rightarrow P)$ the corresponding probability amplitudes. Then $(P \rightarrow R)$ and $(R \rightarrow P)$ are complex numbers. The mappings $(P \rightarrow R) \rightarrow(R \rightarrow P)$ and $(R \rightarrow P) \rightarrow(P \rightarrow R)$ are inverse to each other. By LACK OF INHERENT SPECIALNESS, we may assume they are the same mapping. Such a mapping which is equal to its own inverse, is said to be an involution. The simplest non-trivial involution of the complex numbers is $\alpha \rightarrow \alpha^{*}$. $\left[\alpha \rightarrow \eta \alpha^{*}\right.$ is also an involution for $|\eta|^{2}=\eta \eta^{*}=1\left\{\alpha \rightarrow \eta \alpha^{*} \rightarrow \eta\left(\eta \alpha^{*}\right)^{*}=\eta \eta^{*} \alpha^{* *}=\right.$ $\alpha\}]$. Using Occam's Razor we will assume that $(P \rightarrow R)^{*}=(R \rightarrow P)$ and $(P \rightarrow Q)^{*}=(Q \rightarrow P)$. The simplest way to associate the same positive real number to $(P \rightarrow R)$ and $(P \rightarrow R)^{*}=(R \rightarrow P)$ is to multiply them together to get $(P \rightarrow R)(R \rightarrow P)=(P \rightarrow R)(P \rightarrow R)^{*}=|(P \rightarrow R)|^{2}$. Using Occam's Razor we will assume that $[P \rightarrow R]=(P \rightarrow R)(R \rightarrow P)^{*}=[R \rightarrow P]$.

The probability associated with a complex probability amplitude is equal to the square of its absolute value.
$[R \rightarrow R]=1$.
Returning to our Gedanken Experiment, we have that $[R \rightarrow P]+[R \rightarrow Q]=1$, and the two possibilities $R \rightarrow P \rightarrow R$ and $R \rightarrow Q \rightarrow R$ are indistinguishable. Probability amplitudes should multiply, so $(R \rightarrow P \rightarrow R)=(R \rightarrow P)(P \rightarrow R)=(R \rightarrow P)(R \rightarrow P)^{*}=|(R \rightarrow P)|^{2}$ and $(R \rightarrow Q \rightarrow P)=$
$(R \rightarrow Q)(Q \rightarrow R)=(R \rightarrow Q)(R \rightarrow Q)^{*}=|(R \rightarrow Q)|^{2}$. Thus $(R \rightarrow R)=|(R \rightarrow P)|^{2}+|(R \rightarrow Q)|^{2}=$ $[R \rightarrow P]+[R \rightarrow Q]=1$ and $[R \rightarrow R]=1^{2}=1$.
$[R \rightarrow S]=0$.
In our Gedanken Experiment, the two possibilities $R \rightarrow P \rightarrow S$ and $R \rightarrow Q \rightarrow S$ are indistinguishable, so $(R \rightarrow S)=(R \rightarrow P \rightarrow S)+(R \rightarrow Q \rightarrow S)=(R \rightarrow P)(P \rightarrow S)+(R \rightarrow Q)(Q \rightarrow S)$. This must be equal to 0 . Since $[R \rightarrow P]=[S \rightarrow Q]$, we may assume, using Occam's Razor, that $(S \rightarrow Q)=(R \rightarrow P)^{*}$. Then $(Q \rightarrow S)=(S \rightarrow Q)^{*}=(R \rightarrow P)$. Thus $0=(R \rightarrow P)(P \rightarrow S)+(R \rightarrow Q)(R \rightarrow P)=$ $(R \rightarrow P)[(P \rightarrow S)+(R \rightarrow Q)]$. Thus $(P \rightarrow S)=-(R \rightarrow Q)$ and $(S \rightarrow P)=-(R \rightarrow Q)^{*}$.

If the behavior of a Schrödinger's cat $|R\rangle$ of type $(r, s)$ is given by probability amplitudes $\alpha$ and $\beta$, say that it is of type $(\alpha, \beta)$. Here $\alpha$ and $\beta$ are complex numbers with $|\alpha|^{2}+|\beta|^{2}=1$, $|\alpha|^{2}=r$, and $|\beta|^{2}=s$. Then the Schrödinger's cat $|S\rangle$ which is complementary to $|R\rangle$ is of type $\left(-\beta^{*}, \alpha^{*}\right)$. If $|P\rangle$ is considered to be a Schrödinger's cat relative to $|R\rangle$ and $|S\rangle$, then it is of type $\left(\alpha^{*}, \beta^{*}\right)$.

By behavior, I mean behavior in a Gedanken Experiment. Let $\Omega$ be another observation with values M and N . We can perform the more general Gedanken Experiment which starts by preparing $|\mathrm{R}\rangle$ with $\Psi$, following immediately with the unobserved observation $\Phi$, and then following immediately with the recording $\Omega$. Let $|M\rangle$ be of type $(\gamma, \delta)$ with respect to $\Phi$. What is the type of $|R\rangle$ with respect to $\Omega$ ? The probability amplitude for $R \rightarrow M$ is the sum of the probability amplitudes for $R \rightarrow P \rightarrow M$ and $R \rightarrow Q \rightarrow M$, so $(R \rightarrow M)=(R \rightarrow P \rightarrow M)+(R \rightarrow Q \rightarrow M)=$ $(R \rightarrow P)(P \rightarrow M)+(R \rightarrow Q)(Q \rightarrow M)=(R \rightarrow P)(M \rightarrow P)^{*}+(R \rightarrow Q)(M \rightarrow Q)^{*}=\alpha \gamma^{*}+\beta \delta^{*}$.

This means that the results of any Gedanken Experiment with a preparation followed immediately by a recording are completely determined by the types of the preparation and recording. To be precise, if the preparation is of type ( $\alpha, \beta$ ) and the recording is of type $(\gamma, \delta)$, then the probability amplitude of observing the prepared state $|R\rangle$ in the recorded value $M$ is $\alpha \gamma^{*}+\beta \delta^{\star}$. The probability recorded by the experiment is thus $\left|\alpha \gamma^{*}+\beta \delta^{\star}\right|^{2}$.

## MORE SCHRÖDINGER'S CATS

Using LACK OF INHERENT SPECIALNESS, we conclude, or using Occam's Razor, we assume:

SCHRÖDINGER'S CAT (3): For every pair $\alpha, \beta$ with $|\alpha|^{2}+|\beta|^{2}=1$, there is a Schrödinger's cat of type $(\alpha, \beta)$.

Being a Schrödinger's cat is a condition defined relative to our original observation $\Phi$ with states $|P\rangle$ and $|Q\rangle$. If a Schrödinger's cat of type $(\alpha, \beta)$ is prepared by an observation $\psi$ then, relative to $\Psi,|P\rangle$ is a Schrödinger's cat of type ( $\alpha^{*}, \beta^{*}$ ).

In general, a Schrödinger's cat of "type $(\alpha, \beta)$ relative to $\Phi$ " is also a Schrödinger's cat relative to a Schrödinger's cat of "type $(\gamma, \delta)$ relative to $\Phi$ " unless $\gamma=\mu \alpha$ and $\delta=\mu \beta$ for some $\mu$ with $|\mu|=1$. The type of a Schrödinger's cat of "type $(\alpha, \beta)$ relative to $\Phi$ "
relative to a Schrödinger's cat of "type $(\gamma, \delta)$ relative to $\Phi^{\prime \prime}$ is $\left(\alpha \gamma^{*}+\beta \delta^{*},-\alpha \delta+\beta \gamma\right)$. As expected, $\left|\alpha \gamma^{*}+\beta \delta^{*}\right|^{2}+|-\alpha \delta+\beta \gamma|^{2}=\left(\alpha \gamma^{*}+\beta \delta^{*}\right)\left(\alpha^{*} \gamma+\beta^{*} \delta\right)+(-\alpha \delta+\beta \gamma)\left(-\alpha^{*} \delta^{*}+\beta^{*} \gamma^{*}\right)=$ $\alpha \gamma^{*} \alpha^{*} \gamma^{+} \beta \delta^{*} \beta^{*} \delta+\alpha \delta \alpha^{*} \delta^{*}+\beta \gamma \beta^{*} \gamma^{*}=\left(\alpha \alpha^{*}+\beta \beta^{*}\right)\left(\gamma \gamma^{*}+\delta \delta^{*}\right)=\left(|\alpha|^{2}+|\beta|^{2}\right)\left(\left|\gamma^{2}\right|+|\delta|^{2} \mid\right)=1$

## COMPLEX NUMBERS OF ABSOLUTE VALUE 1

The addition of complex numbers is easy: $(a+b i)+(c+d i)=(a+c)+(b+d) i$. Multiplication seems more mysterious: $(a+b i)(c+d i)=(a c-b d)+(a d+b d) i$. It turns out that multiplication by numbers with absolute value equal to 1 has a simple geometric meaning. The Euclidean distance between two points $\alpha$ and $\beta$ in the complex plane is just $|\alpha-\beta|$. If $|\gamma|=1$, then $|\gamma \alpha-\gamma \beta|=|\gamma(\alpha-\beta)|=|\gamma||\alpha-\beta|=|\alpha-\beta|$. This means that the mapping $\alpha \rightarrow \gamma \alpha$ is a rigid mapping of the plane i.e. it preserves distances. Since $\gamma 0=0$, it must be a rotation or a reflection. Repeating a reflection brings you back to where you started, so if multiplication by $\gamma$ is a reflection, then $\gamma^{2}=1$ and $\gamma=1$ or -1 . If $\gamma=1$, then multiplication by $\gamma$ is a rotation through an angle of 0 , and if $\gamma=-1$, multiplication by $\gamma$ is a rotation through an angle $\pi$. In any case, multiplication by $\gamma$ is a rotation of the complex plane around 0 . Multiplication by $\gamma$ takes 1 into $\gamma, 1 \rightarrow \gamma 1=\gamma$. Multiplication by $\gamma$ is thus a rotation about 0 by the angle that the vector from 0 to $\gamma$ makes with the vector from 0 to 1 . If we think of $\gamma$ and 1 as "being" these vectors, then we can simply say that $\gamma$ is a rotation by the angle between $\gamma$ and 1 . In particular, multiplication by $i$ takes $1 \rightarrow i$ and $i \rightarrow-1$. Multiplication by $i$ is a counterclockwise rotation about 0 of angle $\pi / 2$. This means that for any complex number, ia is perpendicular to $\alpha$ i.e. is rotated by a right angle. Further, if $\alpha$ and $\beta$ are perpendicular, then $\alpha= \pm c i \beta$, where $c=|\alpha| /|\beta|$.

## GAUGE INVARIANCE

Let $\mu$ be a complex number with $|\mu|=1$. Physicists call $\mu$ a "phase factor" or "gauge".

SCHRÖDINGER'S CAT (3'): Schrödinger's cats of type $(\alpha, \beta)$ and of type $(\mu \alpha, \mu \beta)$ are the same. Otherwise, all Schrödinger's cats are different.

Schrödinger's cats being the same means they give the same results in any Gedanken Experiment. If our Gedanken Experiment starts with a preparation of type $(\alpha, \beta)$ and ends with a recording of type ( $\gamma, \delta$ ), then the data recorded by the experiment will be of the form $\left|\alpha \gamma^{*}+\beta \delta^{*}\right|^{2}$. If the experiment started with a preparation of type ( $\mu \alpha, \mu \beta$ ), then we would get $\left|(\mu \alpha) \gamma^{*}+(\mu \beta) \delta^{*}\right|=\mid \mu\left(\alpha \gamma^{*}+\left.\beta \delta^{*}\right|^{2}=|\mu|^{2}\left|\alpha \gamma^{*}+\beta \delta^{*}\right|^{2}=\left|\alpha \gamma^{*}+\beta \delta^{*}\right|^{2}\right.$ i.e. the same result.

If the experiment started with a preparation of type ( $\alpha, \mu \beta$ ) and ended with a recording of type ( $\alpha, \beta$ ), then the recorded data would be $\left|\alpha \alpha^{*}+\mu \beta \beta^{*}\right|^{2}=\left.\left.\left||\alpha|^{2}+\mu\right| \beta\right|^{2}\right|^{2}$. The same experiment starting with a preparation of type $(\alpha, \beta)$ would give $\left||\alpha|^{2}+|\beta|^{2}\right|^{2}$. For $\mu \neq 1$, these are different results.

## A NOTE FOR BUDDHIST AND TIBETAN BUDDHIST READERS

Before moving on from Schrödinger's cat, I should point out to my Buddhist readers that Schrödinger's cat is the meditational object that I promised earlier. Unlike a pot, she is not "acting like she has inherent existence", and so, I would think, is an appropriate subject of a meditation on what "lack of inherent existence" means. For Tibetan Buddhists, Schrödinger's cat might be raised to the status of a yidam, perhaps as an emanation of Vajra Yogini. Vajra Yogini's hand ornaments are the curved knife used in the carnal grounds, which cuts through all obscurations, and the skull cup that holds the ambrosia, the nectar of Emptiness. Schrödinger's cat's hand ornament is, of course, her sharp and precise claws, which cut through all obscurations, and are a manifestation of her sharp and precise mathematics that cuts through all obscurations and are her means of existence. That she does not pretend to "act like she has inherent existence" baths her in the amrita of Emptiness. As we will see, her manifestation in Quantum Mechanics is as "the state space of the quantum bit, the qubit".


## A CHANGE OF PERSPECTIVE

Suppose that a state $|R\rangle$ is prepared with an observation $\Psi$ and immediately observed with an observation $\Phi$ which prepares the states $|P\rangle$ and $|Q\rangle$. There will be probability amplitudes $\alpha$ and $\beta$ with $\alpha$ being the probability amplitude that $|R\rangle$ will be observed to have the value $P$ i.e. $\alpha=(R \rightarrow P)$, and $\beta$ being the probability amplitude that $|R\rangle$ will be observed to have the value $Q$ i.e. $\beta=(R \rightarrow Q)$. By PERSISTENCE (2), this is a complete description of the state $|R\rangle$. Paul Dirac wrote this, using his ket notation, as $|R\rangle=\alpha|P\rangle+\beta|Q\rangle$. Dirac also writes $\alpha=(R \rightarrow P)$ as $\langle P \mid R\rangle$, so that $|R\rangle=\langle P \mid R\rangle|P\rangle+\langle Q \mid R\rangle|Q\rangle$.

If the observation $\Phi$ having the two values $P$ and $Q$ "acts like it has inherent existence",
then it prepares exactly two states, $|P\rangle$ and $|Q\rangle$, and these are the only possible states, Something "has the property P " if it is the state $|\mathrm{P}\rangle$, and "has the property Q " if it is in the state $|Q\rangle$. That is the way that our daily world, our umwelt, works. If $\Phi$ "is not acting like it has inherent existence", it still prepares the two states $|\mathrm{P}\rangle$ and $|\mathrm{Q}\rangle$, but there are many other possible states of the form $\alpha|P\rangle+\beta|Q\rangle$, where $|\alpha|^{2}+|\beta|^{2}=1$. Here, equating "having property $P$ " and "being in the state $|\mathrm{P}\rangle$ " is much more problematic. Following Schrödinger's lead, we have called these new states Schrödinger's cats. Since $\Phi$ has two values, $P$ and $Q$, it is sometimes referred to as being a "bit". The values $P$ and $Q$ are represented by things like + and,- 0 or 1 , or up and down. A bit that is "not acting like it has inherent existence" is said to be a "quantum bit" or just a "qubit". The set of all states $\alpha|P\rangle+\beta|Q\rangle$, where $|\alpha|^{2}+|\beta|^{2}=1$, is said to be the "state space of the qubit" or simply the qubit itself.

While a normal computer manipulates bits, a quantum computer manipulates qubits. At least that is the hope when quantum computers are actually built. The problem is being able to maintain a lot of bits that "are not acting like they have inherent existence" while still being able to manipulate them.

In physics, the most commonly discussed example of a qubit is associated with the quantum mechanical property of "spin". Even though the electron and the photon do not actually "spin", they have a quantum mechanical property that is called spin which acts like a qubit. This spin is observed in any direction in three-dimensional space and gives the value up or down. If observed in a particular direction, this will prepare states |up〉 and $\mid$ down $\rangle$. All other states are of the form $\alpha|u p\rangle+\beta \mid$ down $\rangle$, where $|\alpha|^{2}+|\beta|^{2}=1$. These states correspond to all the states prepared by observations in other directions.

Going back to the relation $|R\rangle=\alpha|P\rangle+\beta|Q\rangle$, the mathematicians think of all the $\alpha|P\rangle+\beta|Q\rangle$, with $\alpha$ and $\beta$ being any complex numbers, as being a "two-dimensional complex vector space" with $|P\rangle$ and $|Q\rangle$ as a "basis". Then the relation $|R\rangle=\alpha|P\rangle+\beta|Q\rangle$ is "expressing $|R\rangle$ in the basis $|P\rangle,|Q\rangle$ ". Also $|R\rangle$ is the expression for $|R\rangle$ in the basis $|R\rangle,|Q\rangle$. Putting that together, when the observation corresponding to $|R\rangle$ is followed immediately by the observation corresponding to $|\mathrm{P}\rangle$ and $|\mathrm{Q}\rangle$, this can be viewed as "changing the basis from $|R\rangle,|S\rangle$ to $|P\rangle,|Q\rangle$.

## A NEW GEDANKEN EXPERIMENT

Consider the Gedanken Experiment with $\Phi$ both as preparation and report, but with the final observation not being immediately after the first observation. In other words, observe with $\Phi$, wait a while, and then observe again with $\Phi$. What happens? For this to be a bonafide Gedanken Experiment, the whole thing must be repeatable. With the same situation set up, and waiting the same amount of time, repetition must lead to the same results. This means that waiting for a given amount of time and then observing with $\Phi$ is a bonafide observation. We can view this new observation, waiting and then observing with $\Phi$, as "changing the basis" i.e. after waiting, $|\mathrm{P}\rangle$ will have changed into a state which is equal to some $\alpha|P\rangle+\beta|Q\rangle$. We can write this as $U(|P\rangle)=\alpha|P\rangle+\beta|Q\rangle$. Then $U(|Q\rangle)=-\beta^{*}|P\rangle+\alpha^{*}|Q\rangle$ and $U(\gamma|P\rangle+\delta|Q\rangle)=\left(\gamma \alpha-\delta \beta^{*}\right)|P\rangle+\left(\gamma \beta+\delta \alpha^{*}\right)|Q\rangle$.

If we show the time dependence by $U_{t}, \alpha_{t}$, and $\beta_{t}$, this tells us how the states change
with time. What can we say about this $U_{t}$ ?

## THE ULTIMATE TRUTH AND SCHRÖDINGER'S EQUATION

In our model, I view the assumption PERSISTENCE as expressing an aspect of The Ultimate Truth. Namely, it is a way that the Welt-an-sich exists in the Umwelt and the Welt-an-sich is the ultimate expression of the Ultimate Truth. However, PERSISTENCE (2) expresses this using probabilities. It says that if something is in the state $|\mathrm{P}\rangle$ defined by $\Phi$, then if it is observed again with $\Phi$ a short time $\Delta t$ later, the probability of getting the value $P$ is very close to 1 and the probability of getting the value $Q$ is very close to 0 i.e. the probability of getting the value $P$ goes to 1 as $\Delta \mathrm{t}$ goes to 0 and the probability of getting the value Q goes to 0 as $\Delta$ t goes to 0 . We have seen that we should express this with probability amplitudes.

PERSISTENCE (3): If something in the state $|P\rangle$ at time $t$ is observed a short time $\Delta t$ later, then there are small complex numbers $\varepsilon_{P}$ and $\varepsilon_{Q}$ such that the probability amplitude to be in the state $|P\rangle$ at time $t+\Delta t$ is $1+\varepsilon_{P}$ and the probability amplitude to be in the state $|Q\rangle$ is $\varepsilon_{Q}$. If $\Delta t$ goes to 0 , then $\varepsilon_{P}$ and $\varepsilon_{Q}$ go to 0 .

Here PERSISTENCE (2) would only say that the absolute value of the probability amplitude to be in the state $|P\rangle$ is close to 1 at time $t+\Delta t$. We can multiply both $\varepsilon_{P}$ and $\varepsilon_{Q}$ by the same "phase factor" to make this probability close to 1 .

Isaac Newton's great mathematical insight was that it was not just important that something went to 0 , it was also important "how" it went to 0 . This was the basis of his Infinitesimal Calculus. I pointed out earlier that Newton's Law of Motion can be viewed through the lens of Occam's Razor as following from the fact that the force and the acceleration being proportional is the simplest relation that has the force and acceleration both 0 at the same time. What is the simplest relation between $\varepsilon$ and $\Delta t$ that has $\varepsilon$ go to 0 when $\Delta t$ goes to 0 ? It is that they are proportional. Using Occam's Razor I will assume that $\varepsilon_{P}$ and $\varepsilon_{Q}$ are both proportional to $\Delta t$. More precisely, this means there are complex numbers $H_{P}$ and $H_{Q}$ such that $\left|\varepsilon_{P}-H_{p} \Delta t\right| /\left|\varepsilon_{P}\right|$ and $\left|\varepsilon_{Q}-H_{q} \Delta t\right| /\left|\varepsilon_{Q}\right|$ go to 0 as $\Delta t$ goes to 0 i.e. $\varepsilon_{P}$ and $\varepsilon_{Q}$ are proportional to $\Delta t$ in the limit as $\Delta t$ goes to 0

PERSISTENCE (4): If something in the state $|P\rangle$ at time $t$ is observed for a short time $\Delta t$ later, then there are complex numbers $H_{P}$ and $H_{Q}$ such that the probability amplitude to be in the state $|P\rangle$ at time $t+\Delta t$ is $1+H_{P} \Delta t$ and the probability amplitude to be in the state $|Q\rangle$ is $H_{Q} \Delta t$.

Now consider the Gedanken Experiment which starts with the preparation of $|P\rangle$ by the observation $\Phi$, has the unobserved observation $\Phi$ at time $t$, and the observation $\Phi$ again at time $t+\Delta t$. Assume that $|P\rangle$ has changed into the state $U_{t}(|P\rangle)=\alpha_{t}|P\rangle+\beta_{t}|Q\rangle$ at time $t$. We can calculate $U_{t+\Delta t}(|P\rangle)=\alpha_{t+\Delta t}|P\rangle+\beta_{t+\Delta t}|Q\rangle$ by adding and multiplying probability amplitudes. By PERSISTENCE (4), there are complex numbers
$H_{P P}, H_{P Q}, H_{Q P}$, and $H_{Q Q}$ such that the probability amplitude of the state $|P\rangle$, at time $t$, being in the state $|P\rangle$, at time $t+\Delta t$, is $1+H_{P P} \Delta t$, the probability amplitude of the state $|P\rangle$, at time $t$, being in the state $|Q\rangle$, at time $t+\Delta t$, is $H_{P Q} \Delta t$, the probability amplitude of the state $|Q\rangle$, at time $t$, being in the state $|Q\rangle$, at time $t+\Delta t$, is $1+H_{Q Q} \Delta t$, and the probability amplitude of the state $|Q\rangle$, at time $t$, being in the state $|P\rangle$, at time $t+\Delta t$, is $H_{Q P} \Delta t$. We have $\alpha_{t+\Delta t}=\alpha_{t}\left(1+H_{P P} \Delta t\right)+\beta_{t} H_{Q P} \Delta t$ and $\beta_{t+\Delta t}=\alpha_{t} H_{P Q} \Delta t+\beta_{t}\left(1+H_{Q Q} \Delta t\right)$. Rearranging gives $\left(\alpha_{t+\Delta t}-\alpha_{t}\right) / \Delta t=\alpha_{t} H_{P P}+\beta_{t} H_{Q P}$ and $\left(\beta_{t+\Delta t}-\beta_{t}\right) / \Delta t=\alpha_{t} H_{P Q}+\beta_{t} H_{Q Q}$. These are the expressions, as $\Delta t$ goes to 0 , for the derivatives of $\alpha$ and $\beta$. We have d $\alpha / d t=\alpha H_{P P}+\beta H_{Q P}$ and $d \beta / d t=\alpha H_{P Q}+\beta H_{Q Q}$. We can combine these and write $d\left(U_{t}(|P\rangle)\right) / d t=\left(\alpha H_{P P}+\beta H_{Q P}\right)|P\rangle+\left(\alpha H_{P Q}+\beta H_{Q Q}\right)|Q\rangle$.

PERSISTENCE (5): If a state $|P\rangle$ evolves over a period of time $t_{0}<t<t_{1}$, then there are functions of $t, H_{P P}, H_{P Q}, H_{Q P}$, and $H_{Q Q}$ such that:

$$
d\left(U_{t}(|P\rangle)\right) / d t=\left(\alpha H_{P P}+\beta H_{Q P}\right)|P\rangle+\left(\alpha H_{P Q}+\beta H_{Q Q}\right)|Q\rangle .
$$

Here $|Q\rangle$ is the state complementary to $|P\rangle$ and $U_{t}(|P\rangle$ is the state that $|P\rangle$ has evolved into at time

Isacc Newton used his infinitesimal calculus to express his Law of Motion in terms of "force". Newton's great rival, Gottfried Leibniz, also invented the calculus [the notation $d g / d x$, and the term "derivative" comes from Leibniz. Newton used the notation $\dot{g}$ and the term "fluxion".] but described motion in terms of "vis viva", or in modern terms "energy". It turns out that in Quantum Mechanics the concept of force doesn't work but the concept of energy does. When scientists use PERSISTENCE (5) to describe how the world evolves, they use the H's to characterize the energy. The H's are thus measured in units of energy while the equation in Persistence (5) is a pure mathematical equation without units. Also, scientists do real experiments with real results that have to be matched by the numerical values of a theory. For these reasons, the scientists "pull out" the factor ( $-\mathrm{i} / \hbar$ ) from the H's. [the -i is so that the H's correspond to what in the classical theory is called the Hamiltonian] [ $\hbar$ is called the reduced Planck constant and equals $1.054571817 \ldots \times 10^{-34}$ joule second.] This result is the famous Schrödinger's Equation.

## SCHRÖDINGER'S EQUATION: iぇdU(|P>) $=\left(\alpha H_{P P}+\beta H_{Q P}\right)|P\rangle+\left(\alpha H_{P Q}+\beta H_{Q Q}\right)|Q\rangle$.

Richard Feynman calls it "the quantum mechanical law for the dynamics of the world".

## TAME SCHRÖDINGER'S CATS OBEY SCHRÖDINGER'S EQUATION.

The task of the physicist in solving the problem he is interested in is to find the Hamiltonian, the H's. The mathematician can then solve the resulting Schrödinger's equation for her. In the spirit of the way that we are doing things, I will consider only the simplest case, namely, when the H's don't depend on time. This is called the "time independent" Schrödinger's equation. I will give a solution in this simple case to give a "taste" of what the solutions to Schrödinger's equation look like.

## SPINNING PHASE FACTORS

Suppose that you are moving at a constant speed s around the unit circle in the complex plane. Let $\alpha=\alpha(t)$, with $|\alpha|=1$, be the complex number you are standing on. This $\alpha$ can also be thought of as being the vector from 0 to $\alpha$ i.e. as the "position vector" of $\alpha$, or, in this case, of you. This position vector is "spinning" at a constant speed $s / 2 \pi$. You also have a "velocity vector" that has length s and points in the direction you are moving. This velocity vector is tangent to the unit circle at the point $\alpha$. This means that it is perpendicular to the position vector $\alpha$ and, if you are moving counterclockwise around the circle, that it is equal to (is)d. [multiplication by i rotates by a right angle i.e. by $\pi / 2$ ] This means that $\alpha$ satisfies the differential equation $d \alpha / d t=$ (is) $\alpha$ i.e. the rate at which $\alpha$ is changing is proportional to $\alpha$ with proportionality constant (is). This is the same relation that describes exponential growth, except that here the proportionality constant is imaginary. For this reason, mathematicians write $\alpha(\mathrm{t})=e^{i s t}$. If you start at the point 1 and move at unit speed, then, since the unit circle has length $2 \pi$, you will arrive back at the point 1 at time $2 \pi$ i.e. $e^{2 \pi i}=1$. Also, $e^{(\pi / 2) i}=\mathrm{i}, e^{\pi i}=-1$, and $e^{(3 \pi / 2) i}=-$. The equation $e^{\pi i}+1=0$ is known as Euler's Identity. Richard Feynman calls it "our gem".

This $e^{i s t}$ is our "spinning phase factor". Let $|\mathrm{R}\rangle=\alpha|\mathrm{P}\rangle+\beta|\mathrm{Q}\rangle$. By Gauge Invariance, we can multiply $|\mathrm{R}\rangle$ by any phase factor and not change anything. By Lack of Inherent Specialness, no particular phase factor is special. We can express this by multiplying by $e^{i s t}$ to get $e^{i s t}|\mathrm{R}\rangle$. This $e^{i s t}|\mathrm{R}\rangle$ is revolving at a constant speed and so doesn't favor any particular phase factor. Also, it introduces a new number, s, which could represent "energy". Of course, by Gauge Invariance, this does nothing. However, if we multiply $\alpha$ and $\beta$ by rotating phase factors with different speeds, then this does make a difference. Occam's Razor suggests this might give a solution to the time-independent Schrödinger's equation. Think of the two "values" P and Q of our observation $\Phi$ as measuring "energy" and being given in units of energy. Let the two phase factors rotate at speeds $P / \hbar$ and $Q / \hbar$, so that $\mathrm{U}_{\mathrm{t}}|\mathrm{R}\rangle=\alpha e^{i(P / \hbar) t}|\mathrm{P}\rangle+\beta e^{i(Q / \hbar) t}|\mathrm{Q}\rangle$, and $\mathrm{d}\left(\mathrm{U}_{\mathrm{t}}\right) / \mathrm{dt}=\mathrm{i}(\mathrm{P} / \hbar) \mathrm{\alpha} e^{i(P / \hbar) t}|\mathrm{P}\rangle+\mathrm{i}(\mathrm{Q} / \hbar) \beta e^{i(Q / \hbar) t}|\mathrm{Q}\rangle=$ $(\mathrm{i} / \hbar)\left[\left(\alpha e^{i(P / \hbar) t}\right) P|\mathrm{P}\rangle+\left(\beta e^{i(Q / \hbar) t}\right) \mathrm{Q}|\mathrm{Q}\rangle\right]$. This is Schrödinger's equation with the $\alpha$ and $\beta$ showing their dependence on $t, H_{P P}=P, H_{P Q}=H_{Q P}=0$, and $H_{Q P}=Q$.*

## HAPPENINGS

We have been viewing the world as being filled with "objects" with "properties" that we "observe." We could also view the world as being composed of "happenings". You throw a ball or get a stomach ache. Of course, in the Welt-an-sich nothing happens, in the sense that "something happening" and "something not happening" can not be distinguished. In an Umwelt, two different happenings are distinguished by an observation. Consider a happening that can happen in two different ways. Each time the happening occurs we can observe which way it occurred. The total number of occurrences is the sum of the numbers for each of the two
possibilities. We can express this by saying that the probability of the happening is the sum of the probabilities for each of the two possible ways it can happen.

If a happening in the Scientific Human Umwelt can happen in two different ways, these two different ways may not be distinguishable. As we have said, then we must use complex-valued probability amplitudes rather than probabilities.

## FUNDAMENTAL THEOREM: Suppose a happening He can happen in two different ways, $\mathscr{H}_{1}$ and $\mathscr{H}_{2}$. <br> If these two ways are distinguishable, then $p\left(\mathscr{H}^{\prime}\right)=p\left(\mathscr{H}_{1}\right)+p\left(\mathscr{H}_{2}\right)$, where $p$ stands for probability. <br> If these two ways are indistinguishable, then $\mathfrak{p ~}(\mathscr{H})=\mathfrak{p}\left(\mathscr{H}_{1}\right)+\mathfrak{p}\left(\mathscr{H}_{2}\right)$, where the $\mathfrak{p}$ are complex valued probability amplitudes with $p(\mathscr{H})=|\mathfrak{p}(\mathscr{H})|^{2}, p\left(\mathscr{H}_{1}\right)=\left|\mathfrak{p}\left(\mathscr{H}_{1}\right)\right|^{2}$, and $p\left(\mathscr{H}_{2}\right)=\left|\mathfrak{p}\left(\mathscr{H}_{2}\right)\right|^{2}$.

In general, if the $\mathscr{H}_{1}$ and $\mathscr{H}_{2}$ are indistinguishable, then probabilities will not add, $\mathrm{p}(\mathscr{H}) \neq$ $p\left(\mathscr{H}_{1}\right)+p\left(\mathscr{H}_{2}\right)$. In this case, it is said that there is "interference". However, if $\mathfrak{p}\left(\mathscr{H} \mathcal{P}_{1}\right)$ and $\mathfrak{p}\left(\mathscr{H _ { 2 }}\right)$, thought of as being vectors, are perpendicular, then the probabilities do add. [In this case $\mathfrak{p}\left(\mathscr{H}_{2}\right)$ $=\operatorname{icp}\left(\mathscr{H}_{1}\right)$ for some real number c , with $\mathrm{p}(\mathscr{H})=|\mathfrak{p}(\mathscr{H})|^{2}=\left|\mathfrak{p}\left(\mathscr{H}_{1}\right)+\mathfrak{p}\left(\mathscr{H}_{2}\right)\right|^{2}=\left|\mathfrak{p}\left(\mathscr{H}_{1}\right)+\mathrm{icp}\left(\mathscr{H}_{1}\right)\right|^{2}=$ $\left|\mathfrak{p}\left(\mathscr{H}_{1}\right)(1+\mathrm{ic})\right|^{2}=\left|\mathfrak{p}\left(\mathscr{H}_{1}\right)\right|^{2}\left(1+\mathrm{c}^{2}\right)$ and $\mathrm{p}\left(\mathscr{H}_{1}\right)+\mathrm{p}\left(\mathscr{H}_{2}\right)=\left|\mathfrak{p}\left(\mathscr{H}_{1}\right)\right|^{2}+\left|\mathfrak{p}\left(\mathscr{H}_{2}\right)\right|^{2}=\left|\mathfrak{p}\left(\mathscr{H}_{1}\right)\right|^{2}+\left|\mathrm{icp}\left(\mathscr{H}_{1}\right)\right|^{2}=$ $\left|\mathfrak{p}\left(\mathscr{H}_{1}\right)\right|^{2}\left(1+\mathrm{c}^{2}\right)$.]

We can use this to view the Fundamental Theorem through the lens of Occam's Razor. Suppose we only know that interference can occur when something can happen in two indistinguishable ways. Suppose we try to find "vector-valued probability amplitudes" that add and multiply properly, tell us what the probabilities are when they can be defined, and, what is new, tell us when there is interference. We want to "encode" the occurrence of interference in the relationship of the vectors $\alpha$ and $\beta$ that are assigned to $\mathscr{H}_{1}$ and $\mathscr{H}_{2}$. The vector assigned to $\mathscr{H}$ will be $\alpha+\beta$. Then $\alpha, \beta$, and $\alpha+\beta$ can be thought of as lying on the sides of a triangle. The Pythagorean Theorem says that $\alpha$ and $\beta$ are perpendicular if and only if $|\alpha+\beta|^{2}=|\alpha|^{2}+|\beta|^{2}$. Thus, if we let $p(\mathscr{H})=\left||\alpha+\beta|^{2}, p\left(\mathscr{H}_{1}\right)=|\alpha|^{2}\right.$, and $p\left(\mathscr{H}_{2}\right)=|\beta|^{2}$, that the $\alpha$ and $\beta$ encode noninterference by perpendicularity is just a reformulation of the Pythagorean Theorem. Occam's Razor now suggests the truth of the Fundamental Theorem. [Actually, there is a little bit more. The vector-valued probability amplitudes should multiply properly. This means that there is a multiplication defined on these two-dimensional vectors for which $|\alpha \beta|^{2}=|\alpha|^{2}|\beta|^{2}$. Then, for $\alpha$ with $|\alpha|=1,|\alpha \beta|=|\beta|$ and the mapping $\beta \rightarrow \alpha \beta$ preserves distances. Since $\alpha 0=0$, it must be a rotation about 0 or a reflection in a line through 0 . A reflection would satisfy $\alpha^{2}=1$ and so $\alpha= \pm 1$. Multiplication by -1 is a rotation through an angle of $\pi$. If $\alpha=1$ is the vector-valued probability amplitude for "nothing happens" and $i$ is the vector-valued probability amplitude that is obtained by rotating 1 by an angle $\pi / 2$, then $i^{2}=i(i)=-1$. Our vectors are actually the complex numbers.]

## PROBABILITY AMPLITUDES REPRESENT NON-INTERFERENCE BY PERPENDICULARITY

Feynman uses this Fundamental Theorem as the basis of his
development of Quantum Mechanics.

## THE TWO-SLIT EXPERIMENT

Imagine a wall in front of you with two slits or maybe nice round holes. Imagine that you throw baseballs or shoot bullets at the wall. Some go through the holes in the wall. Imagine behind the wall a small target that records when it is hit by a ball or bullet. A nice Gedanken Experiment with "happenings". Here we can "observe" which hole the balls or bullets go through by watching with perhaps a slow-motion camera. The two ways of hitting the target are distinguishable and the probabilities add. Now imagine that the slits or holes are much closer together and you are shooting electrons at the wall. Are the two ways of hitting the target still distinguishable?

A basic fact is that light can not be used to distinguish between two objects that are closer together than the wavelength of the light.

Albert Einstein discovered in one of his famous 1906 papers [the one he won the Nobel Prize for] that light comes in individual quanta, photons, that have an energy inversely proportional to the wavelength of the light. A photon of half the wavelength has twice the energy. This means that for sufficiently close holes there will be no ambient photons to distinguish which hole the electron went through.

When there is a way to distinguish which hole the electron went through, there is no interference and the electrons behave just like baseballs or bullets.

When there is no way to distinguish which hole the electron went through, there will be interference for most placements of the target.

Instead of a small target, we could just have a large wall that records where each projectile hits. In this case, if we can distinguish which hole the projectile went through, the pattern on the wall will have a large concentration behind each hole and fewer hits farther away. If we can distinguish which hole the projectile went through, then the wall hits look like a kind of wave pattern with more hits where there is constructive interference and fewer hits where there is destructive interference.

Richard Feynman says that this can be looked at as a kind of general Uncertainty Principle. He shows that the Heisenberg Uncertainty Principle is a consequence of the general Uncertainty Principle [He lets the wall with the holes in it be on rollers and considers the momentum and position of this rolling wall as the projectiles bounce off the edges of the holes.]

We can view this as a form of the Relative Truth: When an observation distinguishes which hole the projectile went through, the projectile acts like it has inherent existence and, when no observation distinguishes which hole the projectile went through, the projectile does not act like it has inherent existence.

We have pointed out that in our theory an observation can only affect events that lie in the future time cone of the observation. If there is an observation on the other side of the wall with the holes, or not, this can not affect what happens before this observation on this side of the wall. An observation would tell us that the projectiles went through one of the holes,
not neither, or both, so this must be true even if no such observation is made. In the experiment, no such observation was made and it is impossible to know which hole the projectile went through. It is not that the projectile somehow didn't go through one and only one of the holes, it is this impossibility of knowing which hole it went through that leads to the interference. For emphasis, it is also not the "not knowing" but rather the "impossibility of knowing" that matters. This "impossibility of knowing" is indistinguishability.

## INVOLUTIONS, FERMIONS, AND BOSONS

A special happening is "nothing happens" i.e. everything stays the same. If "nothing happens" and then "something happens", this is the same as that "something happens". This means that the probability amplitude assigned to "nothing happens" times the probability amplitude assigned to this "something happens" is equal to the probability amplitude assigned to this "something happens". This means the probability amplitude assigned to "nothing happens" is 1.

Another type of special happening is an "involution". An involution is its own inverse e.g. "interchanging two objects". The result of an involution followed by the same involution is the same as the result of "nothing happens". This means that, if a probability amplitude $\varepsilon$ is assigned to an involution, it must satisfy $\varepsilon^{2}=1$. This means that $\varepsilon=1$ or $\varepsilon=-1$. If the involution is "interchanging two identical particles" and $\varepsilon=1$, then the particles are said to be Bosons. If the involution is "interchanging two identical particles" and $\varepsilon=-1$, then the particles are said to be Fermions. In both cases, the identical particles are said to be indistinguishable. Photons are Bosons. Electrons, protons, and neutrons are Fermions.

Suppose something can happen in two different indistinguishable ways which differ by an involution. This means that, if $\alpha$ and $\beta$ are the probability amplitudes assigned to these two indistinguishable happenings, then $\beta=\varepsilon \alpha$ with $\varepsilon=1$ or $\varepsilon=-1$. The probability of the original happening is then $|\alpha+\beta|^{2}=|\alpha+\varepsilon \alpha|^{2}$. If $\varepsilon=1$, then $|\alpha+\beta|^{2}=2\left(2|\alpha|^{2}\right)$. This is twice what the probability would be if the two ways this could happen were distinguishable. If $\varepsilon=-1$, then $|\alpha+\beta|^{2}=0$. There is said to be "total destructive interference" and this "happening" can not happen at all.

This means that Bosons and Fermions behave in quite different ways.
The basic term we have been using is "distinguishable" and it's opposite "indistinguishable". In the Welt-an-sich nothing is distinguishable while in an Umwelt everything is distinguishable. We based our discussion on what it meant for a "state" defined by an observation to be distinguishable. We then discussed when two "happenings" were indistinguishable. We have now defined two identical particles to be indistinguishable if two happenings that differ only by an interchange of these two particles are indistinguishable. This is standard usage in Quantum Mechanics but might be a little confusing in that indistinguishable particles can be distinguished if they are in different states. For example, if two photons don't have identical frequency, direction, and polarization, they can be distinguished, and two electrons that are not at the same location, have the same state of motion, or have the same spin, can be distinguished.

Suppose a Boson enters into a state that already is occupied by an identical Boson.

Then this could also have happened with the two Bosons interchanged, and so the probability of this happening will be twice what it would have been if the two particles were distinguishable. If there were N identical Bosons already in the state, then the probability of another entering is $(\mathrm{N}+1)$ times what it would have been if the particles were distinguishable.

When we discussed a pair of pots that could be either red or blue, there were four possible observations of the pair of pots: red/red, red/blue, blue/red, and blue/blue. If the pots were indistinguishable, there would only be three possible observations: both red, both blue, and one red and one blue. If there are N such pots, if they are distinguishable, there are $2^{N}$ possible observations, and if they are indistinguishable, there are $\mathrm{N}+1$ possibilities.

This indistinguishability of indistinguishable Bosons, that is, that identical Bosons can be in the same state, has many consequences in Quantum Mechanics.

If a Fermian were to enter into a state already occupied by an identical Fermion, then this could happen in the indistinguishable way with the two Fermions interchanged. There would be total destructive interference between these two possibilities and so it would be impossible for the second Fermian to enter the state. This is the famous Pauli Exclusion Principle.

PAULI EXCLUSION PRINCIPLE: Two or more identical Fermions can not occupy the same Quantum state.

Richard Feynman says "What are the consequences of this? ... almost all of the peculiarities of the material world hinge on this wonderful fact. The variety that is represented in the periodic table is basically a consequence of this one rule."

## WHAT IS AN OBSERVATION?

In his 1859 book "Origin of Species", Charles Darwin based his new theory of evolution on the idea of "natural selection". His opponents scoffed. "What an idiot. Nature is not conscious. It does not "select" anything." Darwin's "Bulldog", Thomas Huxley, suggested that he use the phrase "Survival of the Fittest" in the second edition of his work. Darwin's opponents then said "What an idiot. His theory does not say anything. The "fittest" are defined to be those that survive and his theory says that these fittest will survive."

The answer to the first objection is that Darwin was not implying that nature is "conscious". The term "selection" was used in analogy to the way plant and animal breeders "select" which individuals to propagate. To the plant breeder, it is important that she is conscious. To the plant or animal that is not important. It is only important whether they survive and reproduce. Darwin was taking the viewpoint of the plant or animal, not the breeder. If there was no breeder, then nature, and the environment, would provide similar selective forces. The breeder, of course, is breeding these plants and animals for a "purpose", mainly for human usage. Darwin not only assumed that his use of the term "selection" did not imply any "consciousness", but also that it did not imply any "purpose".
[Christians, of course, objected that God was the Breeder and the Purpose was for
the use of Man so that Man could Worship and Glorify Him.]
How am I using the term "observation"? When I am talking about a Gedanken Experiment, then you, the conscious human observer, is making the observation in your human Umwelt or scientific human Umwelt. How would that work when describing what happens on Mars or what it means to say that there is no observation?

Following Darwin, I would like to introduce the term "Natural Observation". This is used in analogy to human observation, but is divorced from "consciousness" and "purpose". A human observation "distinguishes" between things. A Natural Observation "distinguishes between things". A Natural Observation is thus an observed observation in our theory. A human observation, say hearing a sound, is a Natural Observation. So is the movement of the eardrum in response to a sound wave. So is the rustling of a leaf in response to a passing sound wave. In Quantum Mechanics a Natural Observation is said to be a Measurement. Defining in an abstract way exactly what a Measurement is, is known as the "Measurement Problem". It is considered to be very deep and unanswered. In practice, one can simply ignore this problem because one is always considering a special case in which you can determine what constitutes a measurement. This is the same as in Darwin's Theory of Evolution where "fitness" can be defined at the level of the phenotype in each special case e.g. beak shape in Galapagos Finches.

## THE RELATIVE TRUTH FOR THE SCIENTIFIC HUMAN UMWELT

## ANYTHING THAT IS NATURALLY OBSERVED ACTS LIKE IT HAS INHERENT

 EXISTENCE.
## ANYTHING THAT IS NOT NATURALLY OBSERVED DOES NOT ACT LIKE IT HAS INHERENT EXISTENCE.

## EVERYTHING IN A NATURAL UMWELT IS NATURALLY OBSERVED.

This corresponds to what happens in the Double Slit Experiment. Since a Natural Observation is defined to be something that distinguishes and being distinguishable is identified with "acting like having inherent existence", our Relative Truth is almost a tautology. It is saved from being a tautology by the fact that in a special case, we can determine what constitutes a Natural Observation.

In a Gedanken Experiment, it is easy to assume there was no Natural Observation during the experiment except those that we prescribe. In a real experiment, it may be quite difficult to ensure that no spurious Natural Observations occur. This is why we don't have practical Quantum Computers yet.

## SOME FINAL COMMENTS

For those interested in understanding more Quantum Mechanics, I suggest vol. 3
of Feynman's Lectures on Physics. [Feynman's book is available for free download from CalTech] The Feynman Lectures on Physics (caltech.edu)

For those interested in Buddhism, I hope you enjoyed exploring Quantum Mechanics from a Buddhist viewpoint with me and I hope that this helps you in some way with your Buddhist practice.

Evolution by Natural Selection provides you not only an umwelt to live in but also a host of "motivations" built into this umwelt. These "carrots and sticks" are absolutely necessary for your existence, but seeing the "lack of inherent existence" of these "carrots and sticks" is absolutely necessary for your enlightenment, at least according to Buddhism. Seeing the "lack of inherent existence" of "real" carrots and sticks is a start.

Michael Behrens

